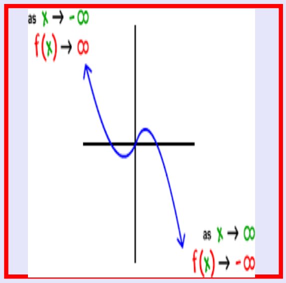
Math 245

Spring 2022

Lecture 34



Consider $P(x) = x^2 - x - 12$

1) Polynomial with degree 2.

2) It has at most 2 Zeros (Solutions $P(x)=0$, Roots)

3) Lead. Coef. = 1 , Constant term = -12

4) List of all possible rational Zeros: $\frac{\pm \text{All Factors of Const.}}{\pm \text{All Factors of L.C.}}$

$$\frac{\pm1, \pm2, \pm3, \pm4, \pm6, \pm12}{\pm1} \Rightarrow \boxed{\pm1, \pm2, \pm3, \pm4, \pm6, \pm12}$$

5) Is 3 a Zero of $P(x)$?

$$\begin{array}{r|rrr} 3 & 1 & -1 & -12 \\ & & 3 & 6 \\ \hline & 1 & 2 & -6 \end{array}$$

Since remainder $\neq 0$, 3 is not a Zero.

6) Is -3 a Zero of $P(x)$?

$$\begin{array}{r|rrr} -3 & 1 & -1 & -12 \\ & & -3 & 12 \\ \hline & 1 & -4 & 0 \end{array}$$

Since Remainder = 0, -3 is a Zero of $P(x)$

7) Factor $P(x)$.

-3 is a Zero of $P(x) \Rightarrow x-(-3)$ is a Factor of $P(x)$.

$P(x) = (x-(-3))(x-4)$ So $P(x) = (x+3)(x-4)$

8) Find all Zeros of $P(x)$.

-3 was a Zero, Now $x-4=0$

$\boxed{x=4}$

4 is also a Zero of $P(x)$.

Consider $P(x) = 3x^2 - 20x + 12$

1) Degree = 2

2) P(x) has at most 2 Zeros.

3) Const. Term = 12, Lead. Coef. = 3

4) List of all possible rational Zeros: $\dfrac{\pm \text{All Factors of Const.}}{\pm \text{All Factors of L.C.}}$

$$\frac{\pm1, \pm2, \pm3, \pm4, \pm6, \pm12}{\pm1, \pm3} = \frac{\pm1}{\pm1}, \frac{\pm2}{\pm1}, \frac{\pm3}{\pm1}, \frac{\pm4}{\pm1}, \frac{\pm6}{\pm1}, \frac{\pm12}{\pm1},$$
$$\frac{\pm1}{\pm3}, \frac{\pm2}{\pm3}, \frac{\pm3}{\pm3}, \frac{\pm4}{\pm3}, \frac{\pm6}{\pm3}, \frac{\pm12}{\pm3}$$

$$\boxed{\pm1, \pm2, \pm3, \pm4, \pm6, \pm12, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}}$$

5) Is $\frac{2}{3}$ a Zero of P(x)?

$$\begin{array}{c|ccc} \frac{2}{3} & 3 & -20 & 12 \\ & & 2 & -12 \\ \hline & 3 & -18 & 0 \end{array}$$

$\frac{2}{3} \cdot 3 = 2$

$\frac{2}{3} \cdot (-18) = -12$

Remainder = 0

$\Rightarrow \frac{2}{3}$ is a Zero of P(x).

6) Factor P(x)

Since $\frac{2}{3}$ is a Zero $x - \frac{2}{3}$ is a factor of P(x)

$$P(x) = \left(x - \frac{2}{3}\right)(3x - 18) = \left(x - \frac{2}{3}\right) \cdot 3(x - 6)$$

$$P(x) = (3x - 2)(x - 6)$$

7) Find all Zeros of P(x).

$3x - 2 = 0$ $\quad \boxed{x = \frac{2}{3}}$

$x - 6 = 0$ $\quad \boxed{x = 6}$

Consider $P(x) = 2x^3 + 3x^2 - 18x + 8$

1) Find degree of P(x). $\boxed{\text{degree} = 3}$

2) How many Zeros? $\boxed{\text{At most 3 Zeros.}}$

3) Lead. Coef. and Constant Term

$\boxed{\text{L.C.} = 2}$, $\boxed{\text{Const. Term} = 8}$

4) List of all possible rational Zeros.

$$\frac{\pm \text{All Factors of Constant Term}}{\pm \text{All factors of L.C.}} = \frac{\pm1, \pm2, \pm4, \pm8}{\pm1, \pm2}$$

$$\boxed{\pm1, \pm2, \pm4, \pm8, \pm\frac{1}{2}}, \cancel{\pm\frac{2}{2}}, \cancel{\pm\frac{4}{2}}, \cancel{\pm\frac{8}{2}}$$

5) Is 4 a Zero of P(x)?

$$\begin{array}{c|cccc} 4 & 2 & 3 & -18 & 8 \\ & & 8 & 44 & 104 \\ \hline & 2 & 11 & 26 & 112 \end{array}$$

$\Rightarrow$ Remainder $\neq 0$

4 is not a Zero of P(x).

6) Is -4 a Zero of P(x)?

$$\begin{array}{c|cccc} -4 & 2 & 3 & -18 & 8 \\ & & -8 & 20 & -8 \\ \hline & 2 & -5 & 2 & 0 \end{array}$$

$\Rightarrow$ Remainder = 0,

-4 is a Zero of P(x)

7) Factor P(x) $\quad P(x) = (x - (-4))(2x^2 - 5x + 2)$

$P(x) = (x + 4)(2x^2 - 5x + 2)$

Maybe -4 is a repeated Zero.

$$\begin{array}{c|ccc} -4 & 2 & -5 & 2 \\ & & -8 & 52 \\ \hline & 2 & -13 & 54 \end{array}$$

-4 is not a repeated Zero.

Is $\frac{1}{2}$ a Solution?

$$\begin{array}{c|ccc} \frac{1}{2} & 2 & -5 & 2 \\ & & 1 & -2 \\ \hline & 2 & -4 & 0 \end{array}$$

$\Rightarrow \frac{1}{2}$ is a Zero of P(x).

Factor P(x) more.

$$P(x) = (x + 4)\left(x - \frac{1}{2}\right)(2x - 4)$$
$$= (x + 4)\left(x - \frac{1}{2}\right) \cdot 2(x - 2)$$

$$\boxed{P(x) = (x + 4)(2x - 1)(x - 2)}$$

3 Zeros $\Rightarrow -4, \frac{1}{2}, 2$

Consider $P(x) = x^4 - 3x^2 - 4$

1) How many Zeros? Why?

$\boxed{\text{at most } 4}$ $\boxed{\text{Degree} = 4}$

2) List of all possible rational Zeros.

$$\frac{\pm \text{All factors of Const.}}{\pm \text{All factors of LC}} = \frac{\pm 1, \pm 2, \pm 4}{\pm 1} = \boxed{\pm 1, \pm 2, \pm 4}$$

3) Find all Zeros.

Try 2 ✓

Missing Term

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & -3 & 0 & -4 \\ & & 2 & 4 & 2 & 4 \\ \hline & 1 & 2 & 1 & 2 & 0 \end{array}$$

$P(x) = (x-2)(x^3 + 2x^2 + x + 2)$

Now try -2 with new factor.

$$\begin{array}{r|rrrr} -2 & 1 & 2 & 1 & 2 \\ & & -2 & 0 & -2 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

$P(x) = (x-2)(x+2)(x^2+1)$

4 Zeros
$\pm 2, \pm i$

Exam 2:
Thursday
April 28th.

x^2+1 has no real Solutions

$x^2 + 1 = 0$

$x^2 = -1 \Rightarrow \boxed{x = \pm i}$